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Price elasticity matrix of demand in power system considering demand response programs

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Abstract. The increasing renewable energy power generations have brought more intermittency and volatility to the electric power system. Demand-side resources can improve the consumption of renewable energy by demand response (DR), which becomes one of the important means to improve the reliability of power system. In price-based DR, the sensitivity analysis of customer's power demand to the changing electricity prices is pivotal for setting reasonable prices and forecasting loads of power system. This paper studies the price elasticity matrix of demand (PEMD). An improved PEMD model is proposed based on elasticity effect weight, which can unify the rigid loads and flexible loads. Moreover, the structure of PEMD, which is decided by price policies and load types, and the calculation method of PEMD are also proposed. Several cases are studied to prove the effectiveness of this method.

1. Introduction

In recent years, the renewable energy power generations are dramatic increasing, due to the increasing pressure of the environment, which brings high random and unpredictable to the power system [1]. Moreover, the increasing power demand and the widening peak-valley differences have seriously affect the safety of power system [2]. With the development of information and communication technology, demand side management (DSM) has become an important way to improve the reliability of power system by interacting between load-side and system-side [3].

Price-based demand response (DR) can lead customers to change their behaviors in power consumption by the leverage of price in power market. The analysis of the regular pattern between customer's power consumption and changing prices is important in the research of DR, which will affect the price setting in power market and the economic benefit of market bodies. Price elasticity of demand is a common measure used in economics to analyze the responsiveness of the quantity demanded of a good or service to a change in its price [4]. Similarly, the response of power consumption to price changing can be expressed as the price elasticity matrix of demand (PEMD). Some studies have focused on the applications of price elasticity matrix [5-10]. The calculations of PEMD are always based on traditional models or determining elasticity coefficient values by experience [6-10]. There is no detailed analysis on the structure and characteristics of PEMD. However, for different price policies and different load types, the structure of PEMD have lots of differences. Some errors will be brought to the forecasting of power consumption by traditional models.

This paper analyzes the influence of price policies and load types on the structure of PEMD. An improved PEMD model based on elasticity effect weight is proposed. The model intuitively reflects the impact of the changing prices on different intervals of electricity consumption. Moreover, several cases are studied to prove the effectiveness of this method. The PEMD can be used in load forecasting and electricity market price setting in a real-time market.

2. Structure of PEMD

2.1. PEMD

In economics, price elasticity of demand refers to the response of demand to price changing. It can be expressed as

$$\varepsilon=\frac{\Delta Q/Q}{\Delta P/P} \tag{1}$$

where ε, Q, P, ΔQ, ΔP are elasticity coefficient, demand, price, demand changes and price changes, respectively. A high ε indicates that demand is highly sensitive to price. $\varepsilon=0$ indicates that price cannot affect demand at all.

Electricity is a special commodity with high timeliness, whose prices can change in real-time and demand can transfer in time. The change of electricity price in each interval will not only affect the electricity consumption in this interval, but also affect the electricity consumption in other intervals. Therefore, the elasticity coefficient can be divided into self-elasticity coefficient and cross-elasticity coefficient [8-10].

The self-elasticity coefficient ε_{ii} expresses the effect of the price changing in the interval i on the electricity consumption in the interval i, which can be defined as

$$\varepsilon_{ii}=\frac{\Delta Q_i/Q_i}{\Delta P_i/P_i} \tag{2}$$

where ΔQ_i and Q_i are demand changes and the original demand in the interval i, respectively. ΔP_i and P_i mean price changes and the original price in the interval i, respectively.

The cross-elasticity coefficient ε_{ij} is defined as the response of the demand in the interval i to the price changes in another interval j, which can be expressed as

$$\varepsilon_{ij}=\frac{\Delta Q_i/Q_i}{\Delta P_j/P_j} \tag{3}$$

Divide the total time T into n intervals and the price elasticity matrix E can be expressed as

$$E=\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1n} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2n} \\ \vdots & \vdots & & \vdots \\ \varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{nn} \end{bmatrix} \tag{4}$$

2.2. *Structure of PEMD*

2.2.1. Influence of price policy For different price policies, the distributions of nonzero elements in price elasticity matrix E are different. This paper chooses two policies as follows.

Policy I: The electricity price of both current interval and subsequent intervals are known, such as day-ahead price. In this policy, the electricity consumption in this interval is related to the price of past intervals, current interval and future intervals. Therefore, every elements of the elasticity matrix may be nonzero.

Policy II : Only the electricity price of current interval is known, such as real-time price. In this policy, the electricity consumption in this interval is related to the price of past intervals and current interval. Therefore, the upper triangular elements of the elasticity matrix are all zeroes, which can be expressed as

$$E = \begin{bmatrix} \varepsilon_{11} & 0 & \cdots & 0 \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{nn} \end{bmatrix} \tag{5}$$

2.2.2. Influence of load type For different load types, the distributions of nonzero elements in price elasticity matrix E are different. This paper divide the power load into two categories: rigid loads and flexible loads.

Rigid loads refer to the loads which cannot be transferred, such as light. Its cross-elasticity coefficients are all zeroes, and the elasticity matrix can be expressed as

$$E = \begin{bmatrix} \varepsilon_{11} & 0 & \cdots & 0 \\ 0 & \varepsilon_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \varepsilon_{nn} \end{bmatrix} \tag{6}$$

Flexible loads refer to the loads that can transfer in some intervals, such as air-conditions and water heaters. The nonzero elements in its elasticity matrix are distributed around diagonal, and the number of nonzero elements in each row is related to the length of the transferred interval D. For example $D = 2$, the elasticity matrix in policy I and policy II can respectively be expressed as

$$E = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 & 0 & 0 \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \varepsilon_{n-1,n-2} & \varepsilon_{n-1,n-1} & \varepsilon_{n-1,n} \\ 0 & 0 & 0 & \varepsilon_{n,n-1} & \varepsilon_{n,n} \end{bmatrix} \tag{7}$$

$$E = \begin{bmatrix} \varepsilon_{11} & 0 & 0 & 0 & 0 \\ \varepsilon_{21} & \varepsilon_{22} & 0 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \varepsilon_{n-1,n-2} & \varepsilon_{n-1,n-1} & 0 \\ 0 & 0 & 0 & \varepsilon_{n,n-1} & \varepsilon_{n,n} \end{bmatrix} \tag{8}$$

3. Improved price elasticity matrix model

3.1. Problems in traditional model

In traditional model, the change of electricity consumption in interval i is generated by the price changing in all intervals from 1 to n . Therefore, the relationship between the change of price and the change of electricity consumption, which is calculated by formula (2) ~ (4), can be expressed as

$$\begin{bmatrix} \Delta Q_1 / Q_1 \\ \Delta Q_2 / Q_2 \\ \vdots \\ \Delta Q_n / Q_n \end{bmatrix} = \frac{1}{n} \cdot E \cdot \begin{bmatrix} \Delta P_1 / P_1 \\ \Delta P_2 / P_2 \\ \vdots \\ \Delta P_n / P_n \end{bmatrix} \tag{9}$$

It means that the price changing in any interval has the same effect on the change of the electricity consumption in interval i, which is $\frac{1}{n}\cdot\frac{\Delta Q_i}{Q_i}$. However, according to the transfer characteristics of loads, the influence of the price changing on the electricity consumption in its own interval should be more significant than that in other intervals. Therefore, the traditional model of PEMD is not accurate.

3.2. Elasticity effect weight

Elasticity effect weight ω_{ij} is defined as an influence effect weight of price changing in interval i on the change of the electricity consumption in interval j. It expressed the proportion of the electricity consumption changing in interval i resulted from the price changing in interval j, which is expressed as

$$\Delta Q_{ij} = \omega_{ij} \cdot \Delta Q_i \tag{10}$$

where ΔQ_{ij} is part of ΔQ_i resulted from ΔP_j.

The electricity consumption in each interval can be split by the length of the transferred interval D.

$$Q_i = \sum_{D=1}^{n} Q_{i,D} \tag{11}$$

where $Q_{i,D}$ is electricity consumption in interval i whose transferred interval is D.

The algorithm of ω under different price policies is shown in Table 1. For policy I, the total length of the transferred time is $2D-1$. For policy II, the total length of the transferred time is D.

Table 1. Algorithm of ω under different price policies.

Policy	Self-elasticity effect weight	Cross-elasticity effect weight
I	$\omega_{ii}=\frac{1}{Q_i}\cdot\sum_{D=1}^{n/2}\frac{1}{2D\text{-}1}\cdot Q_{i,D}$	$\omega_{ij}=\frac{1}{Q_i}\cdot\sum_{D=(\lvert j-i\rvert+1)}^{n/2}\frac{1}{2D\text{-}1}\cdot Q_{i,D}$
II	$\omega_{ii}=\frac{1}{Q_i}\cdot\sum_{D=1}^{n/2}\frac{1}{D}\cdot Q_{i,D}$	$\omega_{ij}=\frac{1}{Q_i}\cdot\sum_{D=(\lvert j-i\rvert+1)}^{n/2}\frac{1}{D}\cdot Q_{i,D}$

Use the elasticity effect weight to correct the elasticity coefficient expressed by formula (2) ~ (3). The improved price elasticity matrix E' can be expressed as

$$E' = \begin{bmatrix} \omega_{11}\varepsilon_{11} & \omega_{11}\varepsilon_{12} & \cdots & \omega_{11}\varepsilon_{1n} \\ \omega_{21}\varepsilon_{21} & \omega_{22}\varepsilon_{22} & \cdots & \omega_{2n}\varepsilon_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \omega_{n1}\varepsilon_{n1} & \omega_{n2}\varepsilon_{n2} & \cdots & \omega_{nn}\varepsilon_{nn} \end{bmatrix} \tag{12}$$

So the relationship between the change of price and the change of electricity consumption in each interval can be expressed as

$$\begin{bmatrix} \Delta Q_1/Q_1 \\ \Delta Q_2/Q_2 \\ \vdots \\ \Delta Q_n/Q_n \end{bmatrix} = E' \cdot \begin{bmatrix} \Delta P_1/P_1 \\ \Delta P_2/P_2 \\ \vdots \\ \Delta P_n/P_n \end{bmatrix} \tag{13}$$

4. Case and discussions

4.1. Data initialization

In order to verify the feasibility of the algorithm, this paper set a scenario as follows. In a certain area, the 30-minutes real-time price (policy II) is implemented to residential customers. If T means one day, then $n = 48$. Table 2 shows the proportion of load types in this area.

Table 2. Proportion of load types.

Transferred interval D	Transferred hours (h)	Proportion	Examples
1	0.50	50%	TV; computer; router
2	1.00	35%	air-condition; refrigerator
8	4.00	5%	water heater; washing machine
16	8.00	10%	EV; battery

For residential customers, the main effect weights affecting the electricity consumption are season, date (workday / weekend) and weather (temperature / humidity). This case chooses the electricity consumption data and electricity price data before and after the implementation of real-time price policy in a summer working day. The maximum temperature is 35℃. The price data are shown in Figure 1. The black curve shows the constant price and the red curve shows the real-time price. The weighted average electricity consumption data are shown in Figure 2. The black curve shows the electricity consumption in constant price and the red curve shows the electricity consumption in real-time price.

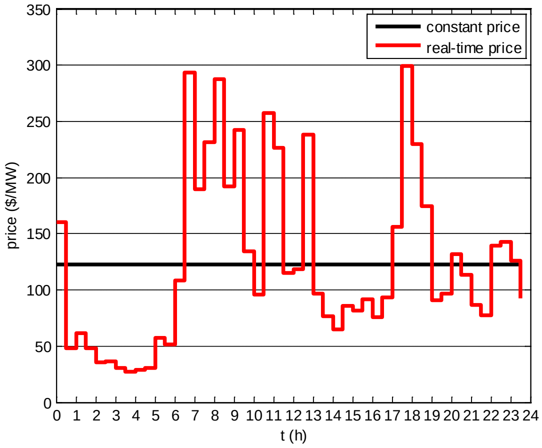

Figure 1. Constant price and real-time price.

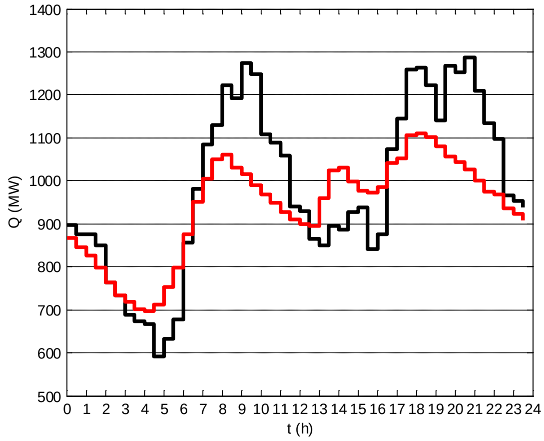

Figure 2. Electricity consumption curve.

0.0551	0.0670	0.0553	0.0471	0.0475	0.0447	0.0432	0.0437	0.0446	0.0634	0.0581	0.2863	−0.0240	−0.0608	−0.0377
0.0564	0.0685	0.0566	0.0482	0.0486	0.0458	0.0442	0.0447	0.0456	0.0649	0.0594	0.2930	−0.0246	−0.0622	−0.0386
0.0940	0.1142	0.0943	0.0804	0.0811	0.0763	0.0737	0.0745	0.0760	0.1081	0.0990	0.4885	−0.0409	−0.1037	−0.0643
0.0969	0.1178	0.0973	0.0829	0.0836	0.0787	0.0760	0.0768	0.0784	0.1115	0.1021	0.5037	−0.0422	−0.1069	−0.0663
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0006	0.0000	−0.0001	−0.0001
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0006	0.0000	−0.0001	−0.0001
−0.0715	−0.0870	−0.0718	−0.0612	−0.0617	−0.0581	−0.0561	−0.0567	−0.0579	−0.0823	−0.0754	−0.3719	0.0312	0.0790	0.0490
−0.0732	−0.0890	−0.0735	−0.0626	−0.0632	−0.0595	−0.0574	−0.0581	−0.0593	−0.0843	−0.0772	−0.3806	0.0319	0.0808	0.0501
−0.0739	−0.0899	−0.0742	−0.0632	−0.0638	−0.0600	−0.0580	−0.0586	−0.0598	−0.0851	−0.0779	−0.3842	0.0322	0.0816	0.0506
−0.3334	−0.4053	−0.3347	−0.2851	−0.2876	−0.2707	−0.2615	−0.2643	−0.2698	−0.3837	−0.3514	0.0000	0.1452	0.3679	0.2281
−0.3119	−0.3792	−0.3132	−0.2668	−0.2691	−0.2533	−0.2446	−0.2473	−0.2524	−0.3590	−0.3288	0.0000	0.1359	0.3442	0.2134
−0.2909	−0.3537	−0.2921	−0.2489	−0.2510	−0.2363	−0.2282	−0.2307	−0.2355	−0.3348	−0.3067	0.0000	0.1267	0.3211	0.1991
−0.0383	−0.0465	−0.0384	−0.0327	−0.0330	−0.0311	−0.0300	−0.0304	−0.0310	−0.0441	−0.0404	−0.1990	0.0167	0.0423	0.0262
0.0503	0.0612	0.0505	0.0430	0.0434	0.0408	0.0395	0.0399	0.0407	0.0579	0.0530	0.2615	−0.0219	−0.0555	−0.0344
0.1214	0.1476	0.1219	0.1038	0.1047	0.0986	0.0952	0.0963	0.0982	0.1397	0.1280	0.6311	−0.0529	−0.1340	−0.0831

Figure 3. Traditional price elasticity matrix.

4.2. Calculation

The traditional price elasticity matrix can be calculated by formula (2) ~ (3). Because of the $n = 48$ matrix is quite large, we select fifteen rows and fifteen columns to show in Figure 3.

The elasticity effect weight matrix can be calculated by formulas in Table 1. The self-elasticity effect weight is the largest as

$$\omega_{ii} = 0.6875 \tag{14}$$

The elasticity effect weight curve for interval $j = 48$ is shown in Figure 4, which expressed that the longer interval between i and 48, the smaller the cross-elasticity effect weight ω_{ij} is.

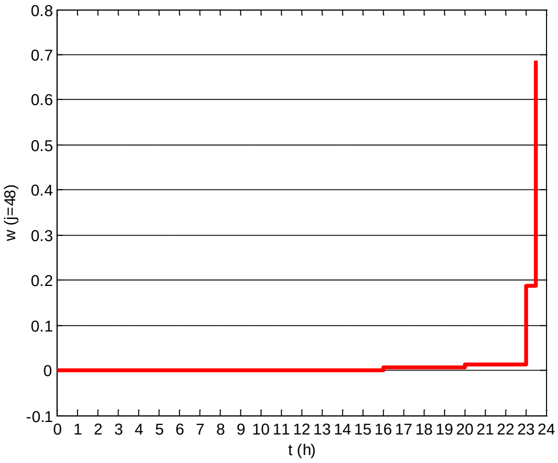

Figure 4. Elasticity effect weight in interval $j = 48$.

The corrected price elasticity matrix can be calculated by formula (12). We also select fifteen rows and fifteen columns to show the result in Figure 5.

-0.0399	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0111	-0.0395	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0008	-0.0109	-0.0403	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0034	-0.0033	-0.0496	-0.1855	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0032	-0.0031	-0.0031	-0.0473	-0.2468	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0030	-0.0029	-0.0029	-0.0029	-0.0628	-0.2108	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0004	-0.0004	-0.0004	-0.0004	-0.0006	-0.0076	-0.1368	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0005	0.0005	0.0005	0.0005	0.0007	0.0007	0.0490	-0.0151	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0006	0.0012	0.0012	0.0012	0.0017	0.0016	0.0079	-0.0099	-0.0921	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0006	0.0006	0.0012	0.0012	0.0017	0.0015	0.0076	-0.0006	-0.0241	-0.0548	0.0000	0.0000	0.0000	0.0000	0.0000
0.0011	0.0011	0.0011	0.0022	0.0031	0.0028	0.0140	-0.0012	-0.0030	-0.0276	-0.0667	0.0000	0.0000	0.0000	0.0000
0.0011	0.0011	0.0011	0.0011	0.0032	0.0029	0.0144	-0.0012	-0.0030	-0.0019	-0.0186	-0.1611	0.0000	0.0000	0.0000
0.0017	0.0016	0.0017	0.0017	0.0024	0.0044	0.0000	-0.0018	-0.0046	-0.0029	-0.0019	-0.0667	-0.1433	0.0000	0.0000
0.0017	0.0017	0.0017	0.0017	0.0025	0.0023	0.0000	-0.0019	-0.0047	-0.0029	-0.0019	-0.0045	-0.0399	0.0000	0.0000
0.0011	0.0010	0.0010	0.0011	0.0015	0.0014	0.0067	-0.0011	-0.0029	-0.0018	-0.0012	-0.0028	-0.0016	-0.2458	0.3959

Figure 5. Corrected price elasticity matrix.

4.3. Discussions

The accuracy of the price elasticity matrix result can be verified by load forecasting. According to the day-ahead prices, the PEMD results can be used to forecast loads. The forecasting results are shown in Figure 6. The black curve shows actual load and the red curve shows forecasting load. The error curve is shown in Figure 7.

Figure 7 shows that the forecasting errors are larger when the prices change small. Because the impact of uncertainty on elasticity coefficient is more significant at that time. The results are not accurate at that interval.

The average error of load forecasting is about 1.40%, which is under the allowable ranges. Therefore, the model is effective and practical.

5. Conclusions

For different price policies and different load types, the structure characteristics of PEMD have many differences. On the basis of the traditional elasticity matrix model, this paper proposes an improved PEMD model by effect weight coefficient with higher accuracy. The case illustrates that the PEMD model has a better application in load forecasting. In the future electricity market, the PEMD will play an important role in the time-of-use electricity pricing and real-time electricity pricing.

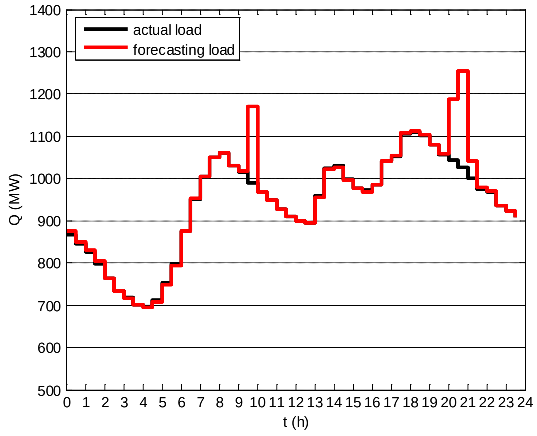

Figure 6. Load forecasting curve.

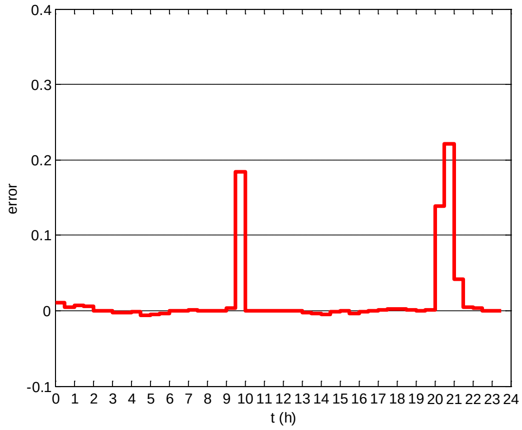

Figure 7. Forecasting error.

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