Two-stage payback model for the assessment of curtailment services provided by air conditioners

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Abstract

Demand-side resources (DSRs) show great potential to help maintain the real-time balance of power system through demand response (DR) programs. However, load payback effect resulted from the control of DSRs poses a threat to the system stability. It is therefore essential to reasonably evaluate payback load so that the benefits of DR will not be over-estimated. Characteristics of load payback effect vary with different types of devices, consumers’ behaviour and control mechanisms. This entails a need to model the payback effect considering different scenarios. This paper focuses on the load payback effect of air conditioners (ACs) for the provision of curtailment services. Two-stage payback model is proposed to characterize the dynamics of payback load during the curtailment period and the recovery period in a more accurate way. Moreover, indexes of capacity payback and energy payback are designed to evaluate the quantity and duration of the payback load. Illustrative studies indicate that the proposed methods can provide reference for selecting control mechanisms, scheduling dispatched ACs and guiding the recovery behaviour of consumers.

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1. Introduction

The integration of demand response (DR) into the power system has recognized the importance of demand-side

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resources (DSRs) to maintain the system balance and also reduce the operation cost [1]. Normally, DSRs participate in the electricity market as curtailment service providers (CSPs) [2]. However, the curtailment operation interrupts the natural diversity of DSRs and therefore gives rise to load payback effect [3], which may greatly harm the system stability and influence the assessment of economic value of DSRs.

Many researches model the payback load through numerical methods. Reference [4] estimates payback load as a ratio of curtailed demand. The payback ratio is determined according to the composition of customer demand, based on which patterns of over-payback and under-payback are clarified. Reference [5] improves this method by considering the time dynamics of payback load through a weighted proportion of previous power consumption. However, the proportion parameters may be hard to achieve in practice. Also, payback duration is not considered in these models, making it difficult to evaluate the effect of payback load comprehensively.

Payback load is affected by the types of devices, consumers’ behaviors and control mechanisms [6]. However, only few researches take into consideration the particularities imposed by different control mechanisms and different curtailment stages. Here we focus on the air conditioners (ACs) for the provision of curtailment services. Direct compressor control mechanism (DCCM) and thermostat set-point control mechanism (TSCM) [7] are two main methods to govern the power consumption of ACs. DCCM is also referred to as on/off control [8], which generates signals to manipulate the on/off status of ACs’ compressor. TSCM changes the temperature set point within the ranges set by consumers. ACs registered to provide curtailment service will curtail demand when receiving the curtailment signal and recover power consumption to the original level when recalled [9]. Different characteristics of payback power during the curtailment and recovery period have to be considered so that the payback model could gain more accuracy.

This paper establishes a two-stage payback model to represent the capacity payback and energy payback during the curtailment period and recovery period of ACs. The dynamics of ACs controlled by DCCM and TSCM are analysed. We find that load payback effect exists both during the curtailment period and the recovery period when ACs are controlled by TSCM. By contrast, DCCM will only lead to load payback during the recovery period and therefore avoid the cost imposed by the payback energy during the curtailment period. Illustrative studies validate the effectiveness of the proposed methods to evaluate the load payback effect quantitatively.

The remainder of this paper is organized as follows. Section 2 introduces the AC load model and control mechanisms. Section 3 establishes the two-stage payback model of ACs for the provision of curtailment services. Illustrative studies are carried out in Section 4 to analyse load payback effect quantitatively with the proposed model. Finally, conclusions are drawn in Section 5.

### Nomenclature

| AC | air conditioner |
| DCCM | direct compressor control mechanism |
| TSCM | thermostat set-point control mechanism |
| CP | capacity payback |
| EP | energy payback |
| RT | recovery time |
| $\Delta T_{set}$ | changes of set point temperature |

### 2. AC load model and control mechanisms

#### 2.1. AC load model

The hybrid state model introduced in [10] is adopted to describe the cyclical operation of individual AC:

$$\frac{d\theta_i(t)}{dt} = -\frac{1}{C_i R_i} [\theta_i(t) - \theta_a(t) + m_i(t) R_i Q_i]$$  \hspace{1cm} (1)

where $\theta_a$ is the ambient temperature, $C_i$ and $R_i$ are the thermal capacity and thermal resistance of the $i$-th AC,
is the energy transfer rate (kJ) of the i-th AC. \( m_i(t) \) represents the on or standby state. It is assumed that the operation state of ACs corresponds to the temperature dead band. For example, ACs are in the cooling mode on summer. Individual AC will switch to on state \( m_i(t)=1 \) when the room temperature reaches its upper band, and similarly, switch to standby state \( m_i(t)=0 \) when the room temperature reaches its lower band.

The relationship between the energy transfer rate and the power consumption of the i-th AC can be expressed with coefficient of performance \( \text{COP}_i \). According to the data in [11], \( \text{COP}_i \) varies mainly with the temperature difference between \( \theta_a \) and \( \theta_r \), which is:

\[
Q_i(t) = p_i \times \text{COP}_i(t) = p_i \times [-\kappa(\theta_a(t) - \theta_r(t)) + \delta] 
\]

(2)

where \( \kappa \) and \( \delta \) are the fitted coefficients of the linear relationship between \( \text{COP}_i \) and \( (\theta_a - \theta_r) \), and \( p_i \) is the consumed power of the i-th AC. Based on the linear fitting results, we set \( \kappa \) to 0.0384/°C and \( \delta \) to 3.9051[11].

2.2. Control mechanisms of ACs for the provision of curtailment services

- **Direct compressor control mechanism[7] (DCCM)**
  
  In the DCCM, ACs change the on/off state of the compressor according to the instruction signal. That means, the dispatched ACs will turn off the compressor upon receiving curtailment instruction and turn on the compressor when recalled.

- **Thermostat set-point control mechanism[7] (TSCM)**
  
  In the TSCM, ACs control their power consumption through changing thermostat set point temperature. For example, if it is in summer and all of the ACs operate in cooling mode, the set point temperature will be increased upon receiving curtailment instruction and decreased when recalled.

3. Two-stage load payback assessment

We define capacity payback and energy payback to reflect the dynamics of payback load. A two-stage payback model is established to characterize the payback load during the curtailment period and recovery period. Denote \( t_{ins} \) as the timestamp when ACs receive the curtailment signal. Typical curves for the dynamics of ACs controlled by DCCM and TSCM are shown in Fig.1.

![Fig.1](image)

**Fig.1** (a) one stage payback of ACs controlled by DCCM (b) two stage payback of ACs controlled by TSCM

Upon receiving curtailment instruction, ACs controlled by TSCM change the set point temperature directly, as is shown in Fig. 1(b). Large numbers of ACs switch from on state to standby state and the aggregate power decreases instantly. When the room temperature reaches its new upper dead band, ACs enter on state, resulting in the increase of aggregate power. This is the payback load on the first stage. Similarly, the dispatched ACs begin to decrease their set point temperature to the original value after receiving the recall signal. Large numbers of ACs switch from standby state to on state, resulting in a sharp increase in aggregate power, which constitutes the payback load on the second stage. By contrast, the aggregate power of ACs is free from payback during the curtailment period controlled by DCCM (Fig. 1(a)), because all the dispatched ACs remain off state. In this case, payback load in the first stage is zero. After receiving the recall signal, all the dispatched ACs are restored to on state and therefore the accumulated power can reach extremely high value.
3.1. Capacity payback

Capacity payback is the maximum value that the aggregate power exceeds its expected level \( P_g^*(t) \). Denote \( CC_g^* \) as the expected curtailment capacity, which is determined by the system operator or the amount previously contracted for. Hence, \( P_g^* \) during the curtailment period is the initial aggregate power consumption \( P_g^0(t) \) minus the expected curtailment capacity. The curtailment process terminates when receiving the recall signal at the timestamp \( t_{rc} \). In this case, \( P_g^*(t) \) is restored to its original value.

\[
P_g^* = \begin{cases} 
P_g^0(t) - CC_g^*, & t_{ins} \leq t \leq t_{rc} \\
P_g^0(t), & t > t_{rc} 
\end{cases}
\]

(3)

Corresponding to the payback load during the curtailment period and recovery period, a two-stage model is established, as illustrated in Fig. 1. Although there is no payback for the ACs controlled by DCCM during the curtailment period, it can be integrated into the two-stage model in which the load payback of the first stage is zero.

- First stage: curtailment period
  \[
  CP_g^* = \max \{ P_g(t) - P_g^0(t) : t_{rs} \leq t \leq t_{rc} \} = \max \{ P_g(t) - P_g^0(t) + CC_g^* : t_{rs} \leq t \leq t_{rc} \}
  \]
  where \( t_{rs} \) is timestamp that the aggregate power has reached \( P_g^0(t) \).

- Second stage: recovery period
  \[
  CP_g^* = \max \{ P_g(t) - P_g^0(t) : t_{rc} \leq t \leq t_{ad} \} = \max \{ P_g(t) - P_g^0(t) : t_{rc} \leq t \leq t_{ad} \}
  \]
  where \( t_{ad} \) is the timestamp that the aggregate power has reached the steady state.

3.2. Energy payback

Energy payback is the additional energy consumption caused by payback load. Energy payback during the curtailment period and recovery period are expressed in shadow area of Fig.1.

- First stage: curtailment period
  \[
  EP_g^* = \int_{t_{rs}}^{t_{rc}} (P_g(t) - P_g^0(t) + CC_g^*) \, dt
  \]
  where \( t_{rs} \) is the time when the aggregate power starts to payback. Since the aggregate power of ACs fluctuates in nature because of its cyclical operation, a threshold (\( \alpha\% \)) is adopted to determine \( t_{rs} \):
  \[
P_g(t_{rs}) = (1 + \alpha\%) \times (P_g^0(t_{rs}) - CC_g^*)
  \]
  (7)

- Second stage: recovery period
  \[
  EP_g^* = \int_{t_{rc}}^{t_{ad}} (P(t) - P^0(t)) \, dt
  \]
  (8)

Apart from recovering aggregate power automatically upon receiving recall instructions, load recovery is also conducted by consumers randomly in many DR programs. Therefore, the recovery behavior may not occur instantly when receiving the recall signal. Recovery time (\( RT \)) is the duration from \( t_{rs} \) to the start of recovery behaviour. If the recovery is conducted by control device of ACs automatically, the recovery time is decided by the response speed of the control device. In this case, the recovery time is an extremely small value. By contrast, if the recovery is conducted by consumers manually, recovery time is decided by the behavior of consumers, which can be highly stochastic. When the recovery time is relatively long, the aggregate power will increase slowly after receiving the recall signal. As a result, the shadow area below the original level of aggregate power is large and the energy payback can be a negative value. Consequently, recovery behavior can be guided to reduce the level of payback.

4. Case studies and discussion

This section quantitatively analyses the load payback effect with different control mechanisms, consumers’ recovery habits and the changes of set point temperature. The influence factors during the curtailment and recovery period are discussed separately based on the proposed two-stage payback model.
4.1. Simulation setup

Assume that ACs aggregate in a residential area. The room area corresponding to the i-th AC unit is \( A_i \), which is subject to normal distribution with the mean value of 30m\(^2\) and the standard deviation of 5m\(^2\). The thermal capacitance \( C_i \) is assumed to equal \((0.04\text{kWh/}^\circ\text{C/m}^2)\times A_i\), and the resistance \( R_i \) is assumed to equal \((0.002\ \text{°C/kW/m}^2)\times A_i\) [10]. Furthermore, the ambient temperature is set to 30°C. The set point temperature is randomly generated between 23°C and 28°C, while the temperature dead band is 2°C. The input power is randomly generated between \((20\text{W/m}^2)\times A_i\) and \((50\text{W/m}^2)\times A_i\). The threshold \( \alpha \% \) is set as 10%. Simulate the dynamics of 5000 ACs when the curtailment duration is one hour.

4.2. Comparison of control mechanisms and consumers’ recovery distribution

Payback load is highly influenced by the diversity of ACs, which includes the distribution of physical parameters, recovery behavior, preference of control methods, etc. Assume that the distributions of physical parameters are certain for a given group of ACs, here we focus on the recovery behavior and its impact on payback load. Recovery time \( RT \) is subject to uniform distribution with the minimum and maximum value of \( a \) and \( b \), which is abbreviated to \( U(a,b) \). Fig.2 illustrates the dynamics of ACs controlled by DCCM and TSCM with various recovery time. Table 1 demonstrates the value of capacity payback and energy payback corresponding to the two-stage model, where \( D \) denotes the ACs controlled by DCCM and \( T \) denotes the ACs controlled by TSCM.

![Fig.2 (a) dynamics of ACs controlled by DCCM (b) dynamics of ACs controlled by TSCM when the changes of set point temperature are 2°C](image)

<table>
<thead>
<tr>
<th>Distribution of recovery time (h)</th>
<th>Capacity Payback (MW)</th>
<th>Energy Payback (MWh)</th>
<th>Recovery period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( CP_D )</td>
<td>( EP_D )</td>
<td>( CP_T )</td>
</tr>
<tr>
<td>( U(0,0.2) )</td>
<td>0</td>
<td>0</td>
<td>2.66</td>
</tr>
<tr>
<td>( U(0,0.4) )</td>
<td>0</td>
<td>0.61</td>
<td>2.55</td>
</tr>
<tr>
<td>( U(0,0.6) )</td>
<td>0</td>
<td>0.61</td>
<td>1.07</td>
</tr>
<tr>
<td>( U(0,0.8) )</td>
<td>0</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>( U(0,1) )</td>
<td>0</td>
<td>0.61</td>
<td>0.059</td>
</tr>
</tbody>
</table>

During the curtailment period, the capacity payback and energy payback only exist with the ACs controlled by TSCM. During the recovery period, the energy payback of TSCM significantly exceeds that of DCCM, while the capacity payback of DCCM is a little bit higher than that of TSCM. Moreover, both the value of capacity payback and energy payback decrease greatly with more scattered distribution of recovery time. Therefore, load serving entities (LSEs) are recommended to sort ACs according to their expected recovery time. The expected recovery time can be obtained from consumers’ curtailment data.
4.3. ACs controlled by TSCM with different changes of set point temperature

When ACs are controlled by TSCM, changes of set point temperature $\Delta T_{set}$ have great impact on the dynamics of aggregate power, as is depicted by Fig.3 (a). Recovery time is assumed to distribute uniformly within the range of 0 to 0.4h. Fig.3 (b) compares the amount of payback on the two stages, in which the blue lines indicate the value of capacity payback, while the red lines indicate the value of energy payback. Payback during the curtailment period and recovery period are indicated by solid marks and hollow marks, respectively.

Fig.3 (b) shows that both the energy payback and capacity payback during the curtailment period is significantly higher than that during the recovery period. Since the temperature dead band of ACs are set as $2^\circ$C, all the dispatched ACs will enter standby state instantly when $\Delta T_{set}$ is larger than $1^\circ$C. Therefore, capacity payback during the curtailment period remains stable when $\Delta T_{set}$ is larger than $1.2^\circ$C, as is illustrated by the blue line with solid marks. During the recovery period, the difference between room temperature and set point temperature is larger when $\Delta T_{set}$ is larger, which will lead to longer duration of on state. In this case, capacity payback increases with larger $\Delta T_{set}$. The red lines demonstrate that energy payback decreases with larger $\Delta T_{set}$ and thus the additional cost imposed by payback energy can be reduced. However, larger $\Delta T_{set}$ will also decrease the comfort level of consumers. Therefore, the payment for consumers and the benefits imposed by larger $\Delta T_{set}$ have to be balanced.

![Fig.3 (a) dynamics of ACs controlled by TSCM with different changes of set point temperature (b) Energy payback corresponding to the changes of set point temperature during the curtailment period and recovery period](image)

5. Conclusion

This paper develops a two-stage payback model of ACs for the provision of curtailment services, based on which the influence factors of curtailment operation and recovery operation can be evaluated separately in a more accurate way. Although the DCCM can avoid payback during the curtailment period, it will lead to higher capacity payback than the TSCM during the recovery period. Larger changes of set point temperature may reduce the amount of energy payback during the curtailment period. However, consumers’ comfort level is reduced and the capacity payback is higher, especially during the recovery period. Moreover, scattered distribution of recovery time will significantly reduce the level of payback. Therefore, consumers’ comfort level, load payback in the two stages and the financial benefits have to be balanced when dispatching ACs with different curtailment preference.

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References