

Distributed Self-triggered Control of Thermostatically Controlled Loads for Providing Ancillary Services

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Abstract—Rapidly growing renewable energies lead to large power fluctuations in power supply, which challenges the secure and stable operation of power systems. In recent years, researchers pay much attention to the regulation of demand-side flexible resources in order to overcome the challenges brought by renewable energies. To provide adequate regulation capacity required by the power system operator, how to aggregate and manage massive demand-side flexible resources is a challenging research problem. This paper investigates on a novel distributed control framework to manage thermostatically controlled loads for providing ancillary services. Compared with existing control methods in literature, the proposed control framework has advantages in terms of much lower computation and communication burden for thermostatically controlled loads. The advantages of the proposed control framework is verified by numerical studies.

Index Terms—thermostatically controlled loads, ancillary services, demand response, distributed self-triggered control

I. INTRODUCTION

Rapidly growing renewable energies lead to large power fluctuations in power supply, which challenges the secure and stable operation of power systems [1]. Generating units are normally deployed in power supply side to provide ancillary services for maintaining the secure and stable operation of power systems facing renewable energy integration. An alternative solution is to exploit regulation potential from demand-side resources in the distribution networks. In recent years, researchers pay much attention to the regulation of demand-side flexible resources in order to overcome the challenges brought by renewable energies [2]. The main approach is to utilize advanced Internet of Things (IoT) technologies

for incorporating massive flexible resources into a unified power control framework [3]. To provide adequate regulation capacity required by the power system operator in the real-time dispatch process, how to aggregate and manage massive demand-side flexible resources is a challenging research problem.

Thermostatically controlled loads (TCLs) are selected as the research object among different kinds of demand-side flexible resources. TCLs account for a large proportion in the total power consumption of the whole society [4], which means the power regulation on TCLs can make enough impacts on the operation of power systems. Besides, the indoor temperature dynamics are much slower than the power control process, which means the thermal comfort of TCL users would not be influenced evidently if the power consumption of TCLs can be regulated properly.

Most popular TCL control framework adopted in literature is the centralized control framework [5], [6]. A centralized controller is responsible for communicating with TCLs and sending TCL power control signals. Considering massive TCLs are aggregated and managed across a wide physical range, much communication burden can be applied on the centralized controller. In addition, the data of TCL users collected by the centralized controller can cause privacy issues. Under this background, this paper alternatively focuses on the distributed control framework incorporating TCLs for providing ancillary services. Compared with the centralized control framework, the main advantages of the distributed control framework are lower communication burden for aggregators and enhanced privacy protection for TCL users [7]. However, the conventional distributed control method relies

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on periodical communication between neighbor TCLs, which can lead to high waste in communication resources [8]. To address this problem, the distributed event-triggered control can be a solution [9], where the transmission of a TCL's control data is determined by a prescribed trigger condition. However, the problem of the distributed event-triggered control is that the local TCL controller needs to continuously check the trigger condition, which obviously increases the computation burden of local TCL controllers [10]. Generally speaking, the distributed event-triggered control for TCLs sacrifices the operation efficiency of local computation resources in order to improve the operation efficiency of local communication resources.

In this paper, a novel distributed control framework is investigated to manage TCLs for providing ancillary services. The proposed distributed self-triggered control method improves both the operation efficiency of local computation resources and communication resources. On the basis of the distributed event-triggered control, the key of the distributed self-triggered control is to compute the next trigger instant instead of continuously checking the trigger condition [11], [12]. Therefore, the main task for the design of the distributed self-triggered control method is how to compute the next trigger instant in an efficient way [13]–[15].

II. MODELING OF THE TCL

For managing TCLs appropriately without violating thermal comfort requirements of users, the relationship between room temperature dynamics and power of a TCL is established. Among different types of commonly used TCLs, the inverter-based TCLs are specifically considered in this paper because their power can be smoothly regulated through variable-speed compressors, compared with traditional on/off switching TCLs [16].

The operation modeling of a TCL is introduced here. On the basis of equivalent thermal parameter model, the temperature variation feature of a room equipped with the TCL is shown as:

$$C_i \frac{dT_i(t)}{dt} = \frac{T_o(t) - T_i(t)}{R_i} - Q_i(t), \quad \forall i \in \mathcal{I}, \quad \forall t \in \mathcal{T}, \quad (1)$$

where C_i and R_i denote air heat capacity and thermal resistance for room i ; $T_i(t)$ and $T_o(t)$ denote indoor and outdoor temperature for room i at time t ; $Q_i(t)$ denotes the cooling capacity of i -th TCL; \mathcal{I} and \mathcal{T} denote sets of TCLs and time. Therefore, (1) describes the relationship between the temperature and the operation of the compressor.

The compressor takes up dominant proportion in TCL's power consumption. The TCL's power consumption and cooling capacity can be approximately expressed as linear with operating frequency of TCL [3], which is shown as:

$$P_i(t) = a_{i1}f_i(t) + c_{i1}, \quad (2)$$

$$Q_i(t) = a_{i2}f_i(t) + c_{i2}, \quad (3)$$

where $P_i(t)$ and $f_i(t)$ denote power consumption and operating frequency of the i -th TCL's at time t . a_{i1} , c_{i1} , a_{i2} and

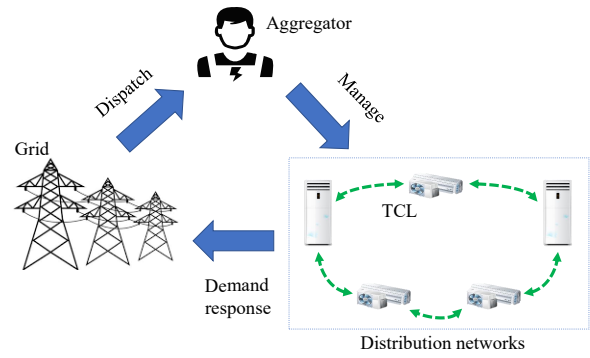


Fig. 1. TCLs for providing ancillary services in distribution networks.

c_{i2} are coefficients related with power and cooling capacity of the i -th TCL. Considering (2) and (3), the relationship between power consumption and cooling capacity of a TCL is shown in (4). The cooling capacity and power of a TCL also shares a linear relationship. Note that for real TCLs, the assumed linear relationship may have very small deviations.

$$Q_i(t) = \frac{a_{i2}}{a_{i1}}P_i(t) + \frac{a_{i1}c_{i2} - a_{i2}c_{i1}}{a_{i1}}. \quad (4)$$

The above four equations build relationship between room temperature and the power of a TCL. Aggregator of TCLs is responsible for controlling the power of TCLs while ensuring the thermal comfort of users. The aggregator mainly manages the total power of TCLs under its control, as:

$$P_{\text{total}}(t) = \sum_{i=1}^N P_i(t), \quad (5)$$

where N denotes the total number of TCLs under the aggregator's control. In summary, the power of a TCL can be smoothly regulated by the compressor speed.

III. DISTRIBUTED SELF-TRIGGERED CONTROL OF TCLS

The overall framework is shown in Fig. 1. The grid operator can send request to aggregators for load management. The aggregator is responsible for managing TCLs in distribution networks for power regulation. TCLs respond to the load control signals and then provide ancillary services to the grid. Note that TCLs are aggregated and managed in a distributed control framework in this paper, where load control signals are transmitted by communication between neighbor TCLs, as shown in Fig. 1.

A. Distributed Event-triggered Control

The distributed self-triggered control of TCLs can be designed based on the event-triggered control. In our previous work regarding the distributed event-triggered control [10], the state of a TCL (the information exchanged between neighbor

TCLs) in the distributed control framework can be designed as:

$$x_i(t) = \begin{cases} \frac{f_i(t) - f_i^0}{f_i - f_i^0}, & \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \text{ Power decrease,} \\ \frac{f_i(t) - f_i^0}{\bar{f}_i - f_i^0}, & \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \text{ Power increase,} \end{cases} \quad (6)$$

where $x_i(t)$ indicates the state of the i -th TCL at time t ; $f_i(t)$ indicates the operating frequency of the i -th TCL's compressor at time t ; f_i^0 indicates the initial operating frequency of the i -th TCL before load control; \underline{f}_i and \bar{f}_i indicate the lower and upper bounds of the operating frequency of the i -th TCL, respectively.

As mentioned in Section I, the transmission of a TCL's control data is determined by a prescribed trigger condition in event-triggered control. Generally speaking, the local controller of the TCL continuously checks the trigger condition, that is, if the detected error surpasses the designed local threshold, the TCL would not send its state value to neighbor TCLs through communication, based on which the communication efficiency can be improved.

The detected error of the i -th TCL is defined as follows:

$$e_i(t) = x_i(t) - \hat{x}_i(t_i^k), \quad t \in [t_i^k, t_i^{k+1}), \quad (7)$$

where $e_i(t)$ denotes the detected error of the i -th TCL at time t ; t_i^k denotes the k -th event instant of the i -th TCL. Besides, the threshold of the i -th TCL is defined as:

$$z_i(t) = \frac{\sigma_i}{4d_i} \sum_{j=1}^N a_{ij} [\hat{x}_j(t_j^h) - \hat{x}_i(t_i^k)]^2, \quad t \in [t_i^k, t_i^{k+1}), \quad (8)$$

where $z_i(t)$ denotes the threshold of i -th TCL; σ_i denotes the threshold coefficient; d_i denotes the number of neighboring TCLs; a_{ij} denotes the element of the adjacency matrix of the communication network; t_j^h denotes the latest h -th event instant for j -th TCL.

Trigger condition for the i -th TCL is expressed as:

$$e_i^2(t) \geq z_i(t). \quad (9)$$

When trigger condition is met, then an event is triggered for the i -th TCL, which means i -th TCL would transmit the information to neighboring TCLs. In the meantime, the control input is updated as:

$$u_i(t) = \sum_{j=1}^N a_{ij} [\hat{x}_j(t_j^h) - \hat{x}_i(t_i^k)], \quad t \in [t_i^k, t_i^{k+1}), \quad (10)$$

where $u_i(t)$ denotes control input of i -th TCL; h denotes the latest event sequence number for the neighbor j -th TCL.

From the observation on the design process of distributed event-triggered control, we can find that the local TCL controller needs to continuously check the trigger condition, which obviously increases the computation burden of local TCL controllers.

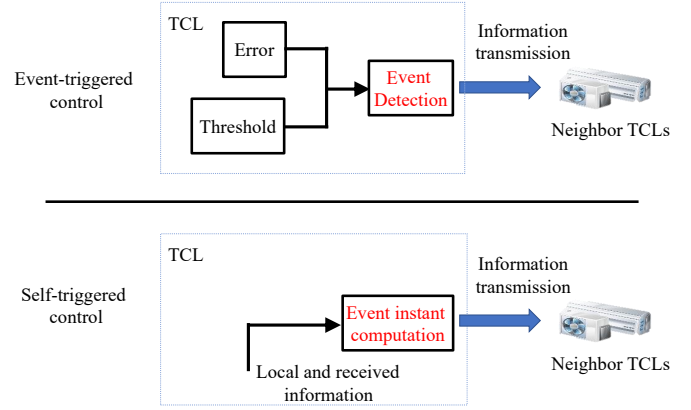


Fig. 2. The comparison between the distributed self-triggered control and distributed event-triggered control for TCLs.

B. Distributed Self-triggered Control

The distributed self-triggered control method is proposed as follows to improve both the operation efficiency of local computation resources and communication resources. The main task for the design of the distributed self-triggered control method is how to compute the next trigger instant in an efficient way. The main difference between the distributed self-triggered control and distributed event-triggered control is illustrated in Fig. 2. It can be noted that the information transmission of a TCL is determined by continuous detection of trigger condition for an event in event-triggered control of TCLs, while the next event instant is computed based on local and received information in self-triggered control of TCLs.

Assuming that at time instant t_i^k , an event is triggered for the i -th TCL, and the threshold $z_i(t)$ and the control input $u_i(t)$ are both updated according to (8) and (10) respectively. It should be noted that unless one of its neighbors is triggered and then transmits the updated value \hat{x}_j , the threshold $z_i(t)$ and the control input $u_i(t)$ would keep constant until the next trigger instant t_i^{k+1} comes.

According to the trigger condition in (9), the following equation is presented in order to compute the next event instant t_i^{k+1} :

$$(x_i(t) - \hat{x}_i)^2 = z_i, \quad (11)$$

where $\hat{x}_i = \hat{x}_i(t_i^k)$. If no information from neighbors is received, the threshold z_i and the control input u_i are constant, and we can compute the next trigger instant t_i^{k+1} as follows.

First, we have

$$x_i(t) = \hat{x}_i \pm \sqrt{z_i}. \quad (12)$$

Note that when $u_i > 0$, we compute the equation as $x_i(t) = \hat{x}_i + \sqrt{z_i}$. when $u_i < 0$, we compute the equation as $x_i(t) = \hat{x}_i - \sqrt{z_i}$.

TABLE I
PARAMETERS

Parameters	Distributions	Parameters	Distributions
a_{i1}	$\mathcal{U}(0.0285, 0.0315)$ kW/Hz	R_i	$\mathcal{U}(1.9, 2.1)$ °C/kW
c_{i1}	$\mathcal{U}(-0.42, -0.38)$ kW	C_i	$\mathcal{N}(390, 78^2)$ kJ/°C
a_{i2}	$\mathcal{U}(0.057, 0.063)$ kW/Hz	T_i^{\max}	$\mathcal{U}(27, 28)$ °C
c_{i2}	$\mathcal{U}(-0.315, -0.285)$ kW	T_i^{set}	$\mathcal{U}(23, 26)$ °C
f_i	$\mathcal{U}(15, 30)$ Hz	T_o	32 °C

¹ \mathcal{U} indicates uniform distributions, and \mathcal{N} indicates normal distributions.

² T_i^{\max} and T_i^{set} indicate users' maximum temperature and initial set temperature.

In addition, according to the design of the TCL's state x_i in (6), due to the limitation of the operating frequency of the TCL (i.e., \underline{f}_i and \overline{f}_i), we have

$$x_i(t) \in [0, 1]. \quad (13)$$

Therefore, when $u_i > 0$, if $\hat{x}_i + \sqrt{z_i} > 1$, then it means no event would be triggered until the neighbors of the i -th TCL send a new state value \hat{x}_j . When a new state value \hat{x}_j is received by the i -th TCL, the computation for the next event instant t_i^{k+1} would be conducted again since the threshold z_i and the control input u_i are both updated. Similarly, when $u_i < 0$, if $\hat{x}_i + \sqrt{z_i} < 0$, no event would be triggered until the neighbors of the i -th TCL send a new state value \hat{x}_j .

Otherwise, when $u_i > 0$, if $\hat{x}_i + \sqrt{z_i} \in [0, 1]$, then the next trigger instant t_i^{k+1} can be calculated as:

$$t_i^{k+1} = t_i^k + \sqrt{z_i}/u_i. \quad (14)$$

Similarly, when $u_i < 0$, if $\hat{x}_i + \sqrt{z_i} \in [0, 1]$, then the next trigger instant t_i^{k+1} can be calculated as:

$$t_i^{k+1} = t_i^k + \sqrt{z_i}/|u_i|. \quad (15)$$

Once a new state value $\hat{x}_j(t_j^{h+1})$ is received by the i -th TCL, the threshold $z_i(t)$ and the control input $u_i(t)$ would be updated, and then the computation for the next event instant is conducted again.

IV. CASE STUDIES

The proposed distributed self-triggered control method for TCLs to provide ancillary services is verified by case studies. TCLs in a community are aggregated and managed by the proposed method to provide ancillary services. The case study parameters are shown in Table I. 2 MW of power cut down is requested for these TCLs. The thermal comfort of users is ensured during the control process.

The case study results are presented as follows. The power consumption cut down of TCLs is shown in Fig. 3, where the request of 2 MW is achieved. The states of TCLs (i.e., the control information exchanged among distributed TCLs) is shown in Fig. 4. The distributed system converges with almost the same control state of the TCLs.

The computation burden comparison between the distributed event-triggered control and proposed self-triggered control for TCLs is shown in Fig. 5. Due to the next trigger instant of the self-triggered control is computed rather than continuously detected, the computation burden of the proposed method can be greatly relieved.

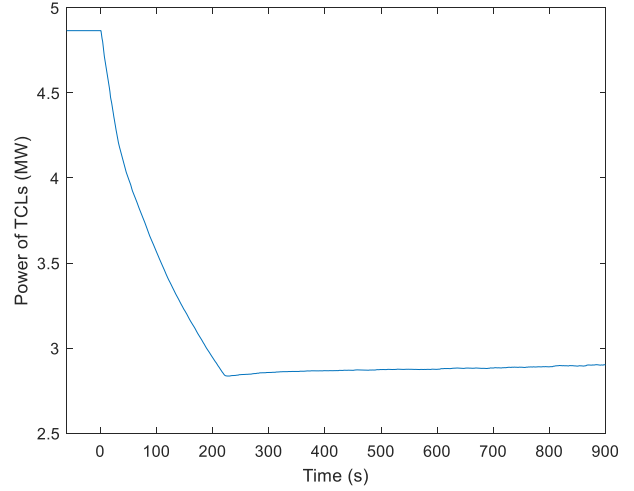


Fig. 3. Power consumption cut down of TCLs.

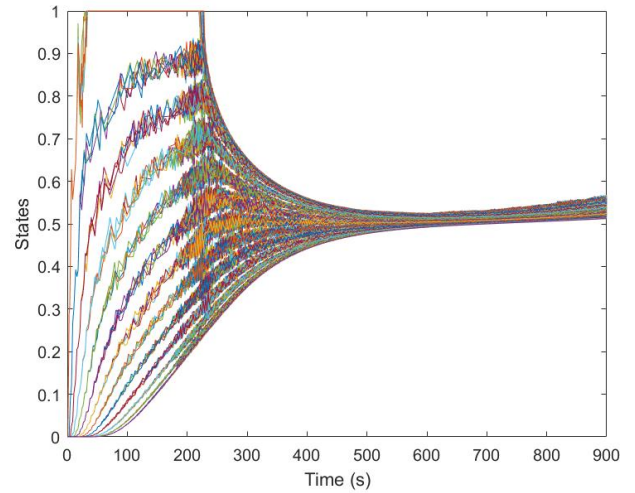


Fig. 4. States of TCLs.

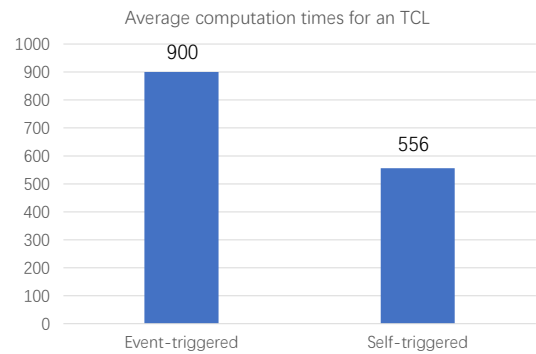


Fig. 5. Comparison of average computation times between the distributed event-triggered control and proposed self-triggered control for TCLs.

V. CONCLUSION

This paper investigates on a novel distributed control framework to manage TCLs for providing ancillary services. Compared with existing control methods in literature, the proposed distributed self-triggered control framework has advantages in terms of much lower computation and communication burden for TCLs. On the basis of the event-triggered control, the key of the distributed self-triggered control is to compute the next trigger instant instead of continuously checking the trigger condition. The advantages of the proposed control framework is verified by numerical studies. Due to the next trigger instant of the self-triggered control is computed rather than continuously detected, the computation burden of the proposed method can be greatly relieved.

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